Spatial Visualization, Visual Imagery, and Mathematical Problem Solving of Students With Varying Abilities

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Abstract

The purpose of this study was to investigate students' use of visual imagery and its relationship to spatial visualization ability while solving mathematical word problems. Students with learning disabilities (LD), average achievers, and gifted students in sixth grade ($N = 66$) participated in this study. Students were assessed on measures of mathematical problem solving, visual imagery representation, and spatial visualization ability. The results indicated that gifted students performed better on both spatial visualization measures than students with LD and average-achieving students. Use of visual images was positively correlated with higher mathematical word-problem–solving performance. Furthermore, the use of schematic imagery was significantly and positively correlated with higher performance on each spatial visualization measure; conversely, it was negatively correlated with the use of pictorial images.

The use of visualization has often been cited as a powerful problem representation process for solving problems (Denis, 1991; Kosslyn & Koenig, 1992; Piaget & Inhelder, 1966). Within the field of mathematics, it has been argued that the use of visual images can be an important help for all sorts of problems, including problems in which nothing geometric is evident (Jencks & Peck, 1972; Polya, 1957; Zimmerman & Cunningham, 1991). Visual imagery, according to Owens and Clements (1998), has a role in establishing the meaning of a problem, channeling problem-solving approaches, and influencing cognitive constructions. Furthermore, it has been suggested that visualization is used to some extent in all reasoning and logic in mathematics (Krutetskii, 1976; Presmeg, 1997). Despite these proposed strengths, several researchers examining the relationship between visualization and mathematical problem-solving performance have found either a weak relationship or no relationship (e.g., Campbell, Collis, & Watson, 1995; Lean & Clements, 1981). One explanation for these inconsistent findings has to do with the various definitions used by the different researchers (Clements, 1982). For example, Clements (1982) defined visual imagery as creating a "picture in the mind" (p. 36) whereas Presmeg (1986a) defined visual imagery as "a mental scheme depicting visual or spatial information" (p. 297). These definitional differences have led to different research foci. For example, some researchers have focused on the vividness of imagery (e.g., Campbell et al., 1995) and others on problem solvers' preferences in using visual imagery (e.g., Lean & Clements, 1981).

It has been suggested that all mathematical tasks require spatial thinking (Fennema, 1979). Furthermore, spatial ability has been found to be positively correlated with measures of mathematics performance (Battista, 1990; Clements & Battista, 1992; Fennema & Sherman, 1977) and noted as being a significant factor in specific areas of mathematics, such as geometry, and in solving problems, in particular complex problems (Burnett, Lane, & Dratt, 1979; Grobecker & De Lisi, 2000; Kaufmann, 1990). The strength of the relationship between visualization and spatial ability, however, has been widely debated (Krutetskii, 1976; Lean & Clements, 1981). Again, one possible explanation may relate to how spatial ability is defined and, subsequently, assessed (Clements, 1982). Despite the varying definitions of "spatial ability," definitions of spatial factors, such as spatial visualization, appear to make use of visual imagery. For example, one definition of spatial visualization—the definition that is also used in this study—is the "ability to mentally manipulate, rotate, or twist, or invert a pictorially presented stimulus object" (McGee, 1979, p. 893). It has been suggested that mental imagery is a type of spatial ability (Clements, 1982). Conversely, spatial ability has been suggested to be linked to visual imagery (Presmeg, 1986a).

Integrating Spatial Ability, Visual Imagery, and Mathematical Performance

Recent research has moved from viewing imagery as being "general and undifferentiated" (Kozhevnikov, Hegarty, & Mayer, 2002, p. 48) to being composed of different components. Using cognitive psychology and neuroscience research (e.g., Baddeley, 1992; Baddeley & Lieberman, 1980; Farah, Hammond, Levine, & Calvanio, 1988; Jonides & Smith, 1997; Kosslyn & Koenig, 1992) suggesting that imagery is composed of two distinct visual and spatial components, Kozhevnikov et al. (Hegarty & Kozhevnikov, 1999; Kozhevnikov et al., 2002) have suggested that two different types of visual images exist: visual imagery and spatial imagery. *Visual imagery* refers to the representation of the visual appearance of an object, such as its shape, color, or brightness, whereas *spatial imagery* refers to the representation of the spatial relationships between the parts of an object and the location of objects in space or their movement. Additional support for variation in imagery types has also come from researchers in education. Presmeg (1986a, 1986b), for example, identified five types of visual imagery that may be used by students to solve mathematical problems.

Focusing on specific imagery types has provided new insight into the relationships between spatial ability, visualization, and mathematics problem-solving performance. Presmeg (1997) noted that all imagery types have some role and function in mathematical problem solving; she considered pattern imagery (i.e., imagery where pure relationships are depicted in a visual–spatial scheme) to be the most essential because it shows the relational aspects of a problem and, thus, seems better suited to abstraction and generalization. Hegarty and Kozhevnikov (1999) found that imagery generated by sixth-grade students to solve mathematical word problems could be reliably classified as being either *pictorial* imagery (i.e., images that encode the visual appearance of objects or persons described) or *schematic* images (i.e., images that encode the spatial relations described in a problem). Furthermore, they found that the use of schematic images was positively related to success in mathematical problem solving, whereas the use of pictorial images was negatively related to success in mathematical problem solving. Finally, the use of schematic imagery was associated with high spatial visualization ability. Kozhevnikov et al. (2002), in three studies involving 60 college students, identified two types of visualizers: those who have high spatial ability, and those who have low spatial ability. High spatial visualizers (spatial type) engaged spatial–schematic imagery systems to solve problems, whereas low spatial visualizers (iconic type) engaged visual–pictorial imagery systems to solve problems.

Learning Disabilities, Visual Imagery, and Mathematical Problem Solving

Only a few studies have investigated the use of visual imagery by students with learning disabilities (LD) to solve mathematical word problems. Montague, Bos, and Doucette (1991) found that eighth-grade students with LD differed significantly from both high- and average-achieving students in the quality of problem representation strategies. Interviews revealed that high- and average-achieving students typically imagined themselves in the problem or drew a picture if the problem was difficult. In contrast, students with LD most frequently responded that they did not use visualization to help them solve problems.

In a study by van Garderen (2002; see van Garderen & Montague, 2003), 66 sixth-grade students representing three levels of problem-solving ability—students with LD, average-achieving (AA) students, and gifted students—were interviewed and observed while they were solving mathematical word problems to determine what types of visual images the students of varying ability typically used and how their visual imagery use related to their mathematical word problem solving performance. As with Hegarty and Kozhevnikov (1999), the visual images that were generated by the students could be classified as being either primarily pictorial or primarily schematic. Furthermore, schematic imagery was positively correlated with success in mathematical problem solving, whereas pictorial representations were negatively related to success in mathematical problem solving. Moreover, van Garderen (2002) also found that differences in imagery use existed among the different groups. First, gifted students used visual images significantly more often than students with LD, supporting other research that students with LD typically use fewer representation strategies (e.g., paraphrasing and visualization) while solving problems (Montague & Applegate, 1993a, 1993b). Second, gifted students used schematic imagery—a more sophisticated type of imagery—significantly more often than students with LD and AA students. Third, students with LD used pictorial imagery more often than gifted students. These findings were considered problematic, given that schematic representations were used on correctly solved problems approximately 76% of the time, whereas pictorial representations were used approximately 70% of the time for incorrect solutions. These findings provided additional support for past research suggesting that the strategies that students with LD use to solve mathematical problems differ qualitatively from the strategies used by their peers (e.g., Montague & Applegate, 1993a, 1993b, 2000; Swanson, 1989; Swanson, Carson, & Sachse-Lee, 1996).

Van Garderen and Montague (2003) did not report findings related to spatial ability. Hegarty and Kozhevnikov (1999) found, as previously stated, that high spatial visualization ability was positively and significantly correlated with the use of schematic imagery. Although they did not find a significant correlation between pictorial imagery use and spatial visualization ability, there was a negative trend. Interesting enough, no significant correlations between spatial relations and either type of imagery were found. A limitation acknowledged in their study was that the sample only consisted of boys. Although this was not considered a concern, as an earlier study with male and female college students suggested that gender was not an issue, the generalizability of the results was questionable and in need of further empirical testing. Moreover, their study did not include students of varying abilities. Therefore, the purpose of this article is to present the findings of an investigation of performance on spatial visualization tasks by students with LD, average-achieving students, and gifted students. Furthermore, findings on the relationships between spatial visualization ability, visual-imagery type use, and overall mathematical problem-solving performance are also presented. The following questions guided this research:

1. Are there differences among sixth-grade students with LD, average-achieving students, and gifted students on measures of spatial visualization ability?
2. What is the relationship between spatial visualization ability and mathematical problem-solving performance?
3. What is the relationship between spatial visualization ability and visual–spatial representation type?

Method

Participants

Sixth-grade students in four urban south Florida elementary and middle schools participated in this study. The students ($N = 66$) represented three levels of problem-solving ability: students with learning disabilities (LD), average-achieving (AA) students, and gifted (G) students. Students with LD, in addition to meeting district eligibility criteria (see Appendix A), had a full scale score of 85 or more on the *Wechsler Intelligence Scale for Children–Revised* (WISC-R; Wechsler, 1976). Average-achieving students were nominated by their teachers as performing at an average level in their math class. Gifted students, in addition to meeting district eligibility criteria (see Appendix A), had a full scale score of at least 130 on the WISC-R. All students were English dominant as determined by school records.

Informed consent forms were distributed to the students with LD and G students who qualified to participate and to approximately 40 AA students. Due to time constraints to complete the study before the end of the academic year, the first 22 students in each group who returned a signed form participated. To determine differences in math achievement, the *Woodcock-Johnson Tests of Achievement*, third edition (WJ-III ACH; Woodcock, McGrew, & Mather, 2001), Calculation, Math Fluency, and Applied Problems subtests were administered. One-way ANOVAs were conducted to determine if group differences existed on the WJ-III ACH subtests and Broad Math Ability. Standard scores were used in the analyses. Significant group differences were found for WJ-III ACH Calculation, $F(2, 63) = 44.22$, $p < .001$, $\eta^2 = .58$; Math Fluency, $F(2, 63) = 23.97$, $p < .001$, $\eta^2 = .43$; and Applied Problems, $F(2, 63) = 40.71$, $p < .001$, $\eta^2 = .56$; and Broad Math, $F(2, 63) = 57.60$, $p < .001$, $\eta^2 = .65$. Tukey's post hoc analyses for Calculation, Math Fluency, and Broad Math indicated that G students scored significantly higher than AA students and students with LD, and AA students scored significantly higher than students with LD (LD < AA < G). Tukey's post hoc analyses for Applied Problems indicated that the G students scored significantly higher than AA students and students with LD (LD = AA < G). Means and standard deviations along with demographic data are presented in Table 1.

Measures and Scoring

Mathematical Problem Solving Performance and Visual Imagery Use. In addition to assessing mathematical problem solving performance using the WJ-III ACH Applied Problems subtest (Woodcock, McGrew, & Mather, 2001), mathematical word-problem–solving performance was examined using the *Mathematical Processing Instrument* (MPI). This instrument was developed and used by Hegarty and Kozhevnikov (1999) in their research with sixth-grade students without disabilities. The MPI consisted of 13 mathematical word problems that required some form of analysis and thought to solve them. The Kuder-Richardson Formula 20 (KR20) internal consistency reliability estimate for the MPI in this study was .85. Each problem was printed on a card and read to students individually. The MPI was also used to determine visual imagery use. After the student solved each problem, the student was interviewed about how they solved it and whether a visual image had been used.

Four different measures were scored from the responses on the MPI. The first score was the number of problems solved correctly. Second, whether the student used a visual image while solving each problem was recorded. For each problem the student did *not* use a visual image, a score of 0 was given. A score of 1 was given on each problem for which the student used a visual image. The third and fourth scores, based on Hegarty and Kozhevnikov's (1999) study, measured the extent to which the students' visual images were either pictorial or schematic. A visual image was scored as *primarily pictorial* if the student reported or drew an image of the objects or persons referred to in the problem. For example, consider the responses to the following problems:

Problem 1: The diameter of a can of peaches is 10 units. How many cans will fit in a box 30 units by 40 units (one layer only)?

Problem 2: On one side of a scale, there are three pots of jam and a 100-ounce weight. On the other side, there is a 200-ounce and a 500-ounce weight. The scale is balanced. What is the weight of one pot of jam?

The following imagery reported in Problems 1 and 2, respectively, was scored as primarily pictorial:

All I see is like, a little can of peaches and, like, a lot of other cans.

A weight balance with pots of jam on each side.

A number of diagrams drawn by the students were categorized as being primarily pictorial. A sample diagram for responses to Problems 1 and 2 that were scored as primarily pictorial is shown in Figure 1.

A visual image was scored as *primarily schematic* if the student drew a diagram, used gestures showing the spatial relations between objects in a problem in explaining the solution strategy, or reported a spatial image of the relations expressed in the problem. The following responses for Problems 1 and 2, respectively, were scored as being primarily schematic imagery:

> I did 10 into 30 and 10 into 40 and I added, no, multiplied them. [Probe: *Did you see a picture of the problem in your mind while you were solving it?*] It was a box and it said 30 and 40 on each side and there were cans going across that said 10 and vertical.

> There was a scale balanced with a 500 and 200 weight on one side and 100 ounce and 3 pots of jam on the other side.

A sample diagram drawn for responses to Problems 1 and 2 that were scored as being primarily schematic is shown in Figure 2.

The author and a research assistant, who was blind to the hypotheses of the study, coded all student responses independently. Interrater agreement was determined by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100 (Kazdin,

TABLE 1
Demographic Data and Mathematical Achievement Scores of Participants by Group

Variable	LD	AA	G
Age (years)			
M	12.06	11.87	11.95
SD	0.48	0.50	0.39
Gender			
Male	14	9	11
Female	8	13	11
Ethnicity			
European American			
n	6	4	20
%	27.3	18.2	90.0
Hispanic			
n	9	11	2
%	40.9	50.0	9.1
African American			
n	7	5	0
%	31.8	22.7	0.0
Asian American			
n	0	2	0
%	0.0	9.1	0.0
Free/Reduced Lunch			
Yes			
n	9	10	0
%	40.9	45.5	0.0
No			
n	13	12	22
%	59.1	54.5	100.0
WISC-R Full Scale			
M	99.18	—	136.60
SD	9.76	—	5.40
WJ-III ACH[a]			
Calculation			
M	91.59	103.23	120.64
SD	12.73	8.92	8.79
Math Fluency			
M	89.27	99.27	116.45
SD	13.70	11.75	13.96
Applied Problems			
M	90.05	96.32	116.32
SD	10.32	8.48	11.26
Broad Math			
M	88.41	98.91	121.36
SD	11.60	8.43	10.92

Note. *N* = 22 for all groups. LD = students with learning disabilities; AA = average achievers; G= gifted students; WISC-R = *Wechsler Intelligence Scale for Children–Revised* (Wechsler, 1976); WJ-III ACH = *Woodcock-Johnson Tests of Achievement,* 3rd ed. (Woodcock, McGrew, & Mather, 2001).
[a]Entries for WJ-III ACH are grade-referenced standard scores (*M* = 100; *SD* = 15).

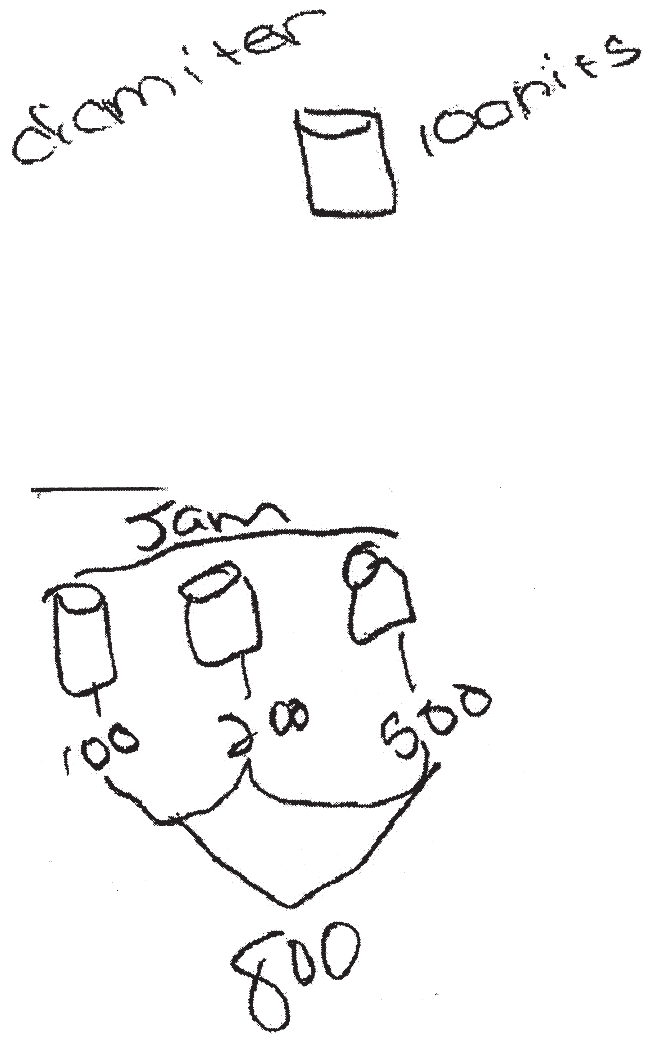

FIGURE 1. Examples of primarily pictorial images produced for Problems 1 and 2. Note how the use of a pictorial image for Problem 2 leads to a wrong answer.

1982). Overall, initial agreement for rating the protocols was 83%. Disagreements were resolved through discussion, for a final interrater agreement of 100%.

Spatial Visualization Ability. Two measures were selected to measure spatial visualization ability: the Block Design subtest from the *Wechsler Intelligence Scale for Children*, third edition (WISC-III; Wechsler, 1991), and the *Middle Grades Mathematics Project Spatial Visualization Test* (MGMP-SVT; Lappan et al., 1981). For the Block Design subtest, students were given two-dimensional geometric patterns that they were to replicate using cubes. The total raw score, calculated according to procedures outlined in the manual, was used for all statistical analyses.

The MGMP-SVT is an untimed test comprising 32 multiple choice items. The items consist of views of one-, two-, and three-dimensional figures shown in a line drawing. The students were to identify a rotated view of the figure in a dimension different from the given view. Scoring of this measure consisted of rating the multiple choice responses as either correct or incorrect. A total score was calculated by summing the correct responses. Because of a negative item total correlation, Items 22 and 24 were removed from the statistical analyses, for a total possible score of 30. The reliability coefficient (KR20) for the measure was .83.

Procedure

All measures were administered individually to the students. The measures were administered during two sessions, each session between 45 and 60 min long. During the first session, the WJ-III ACH Calculation, Math Fluency, and Applied Problems subtests and the MGMP-SVT were administered. Prior to answering the problems on the MGMP-SVT, the students were given two sample items to practice. The WISC-III Block Design subtest and the MPI with the interview were administered during the second session. Where appropriate, the examiner read all the questions to the student. Before the MPI was presented, the goals of the interview were first explained to each student by reading the following:

> I am interested in how children think while solving mathematical word problems. I am going to ask you to solve 13 math problems. You may solve these problems on the paper that I have given to you. I will ask you several questions after you solve each problem. There are no wrong or right answers to my questions. I am interested only in how you solved the problems.

Data Analyses

The dependent variables in this study were (a) the MPI test score, (b) the WJ-III ACH Applied Problems score, (c) the MPI visual imagery use score, (d) the MPI pictorial imagery score, (e) the MPI schematic imagery score, (f) the WISC-III Block Design subtest score, and (g) the MGMP-SVT score. A one-way ANOVA was employed to detect differences among groups on total scores of the WJ-III ACH subtests and Broad Math score, MPI test score, visual imagery use, visual imagery types, WISC-III Block Design subtest, and the MGMP-SVT. For all ANOVAs, Tukey's post hoc procedure was used to determine the nature of the differences. Furthermore, effect sizes were calculated. Cohen's (1988) benchmarks for η^2 were used to determine effect size (.01 is considered a low effect size, .06 a medium effect size, and .14 a high effect size). Pearson product-moment correlation coefficients were employed to determine the relationships between spatial visualization ability and mathematical word-problem–solving performance on the MPI, visual imagery use and type, and spatial visualization ability.

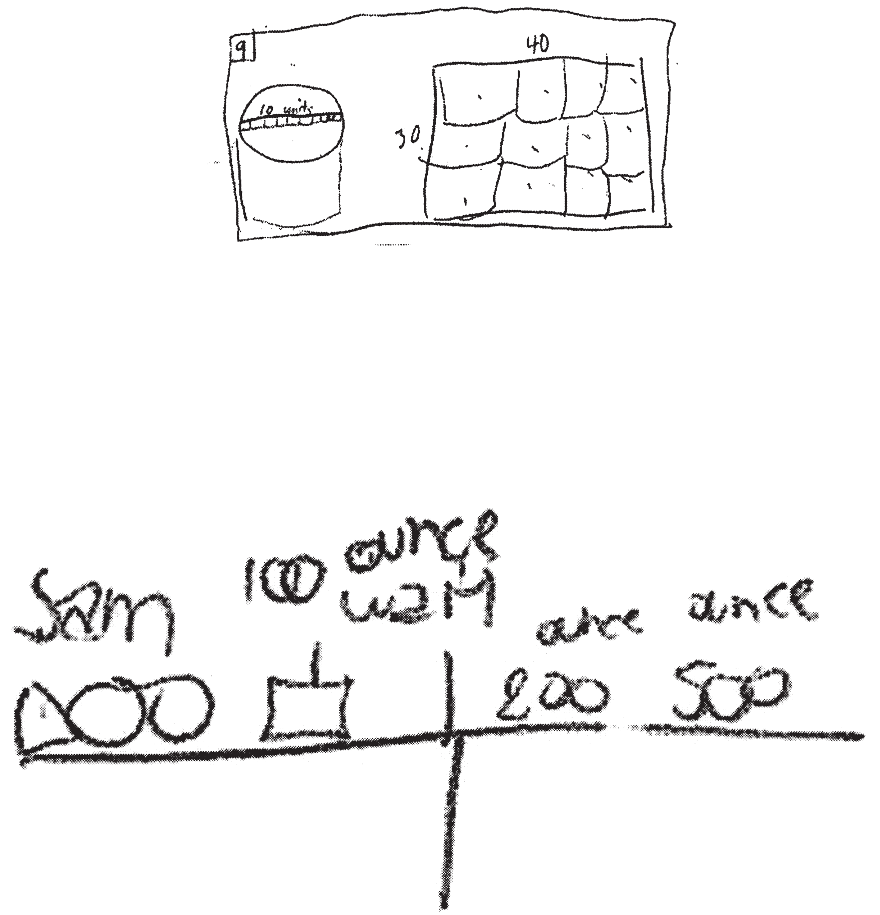

FIGURE 2. Examples of primarily schematic images produced for Problems 1 and 2.

Results

Spatial Visualization Ability

Spatial visualization ability was measured by two measures: the Block Design subtest from the WISC-III (M = 41.56, SD = 14.56) and the MGMP-SVT (M = 11.67, SD = 5.78). Using ANOVA, a significant difference was found for the Block Design subtest, $F(2, 63) = 7.09$, $p < .01$, $\eta^2 = .18$, and the MGMP-SVT, $F(2, 63) = 15.643$, $p < .001$, $\eta^2 = .33$. For both measures, Tukey's post hoc test revealed significant differences between students with LD and G students, and between AA students and G students. No significant difference was found between students with LD and AA students. Gifted students scored higher than AA students and students with LD. Means and standard deviations on these measures are presented in Table 2.

Spatial Visualization Ability and Math Problem-Solving Performance

On the MPI, overall, the students solved less than half the problems correctly (4.8 problems out of the 13 problems possible). Using ANOVA, a significant effect for group was found, $F(2, 63) = 42.58$, $p < .001$, $\eta^2 = .58$. Tukey's post hoc revealed a significant difference between G students and students with LD and between G students and AA students. No significant difference was found between students with LD and AA students. Gifted students scored higher than students with LD and AA students on the MPI. Means and standard deviations on these measures are presented in Table 2.

The relationships between each spatial visualization ability measure and mathematical problem-solving performance on the MPI and the WJ-III ACH Applied Problems subtest showed significant positive correlations (see Table 3). Higher performance on each spatial visualization measure was associated with higher performance on the MPI. Although not all correlations were statistically significant, spatial visualization ability measures and mathematical problem-solving performance on the MPI were positively correlated by group (see Table 4). Positive correlations by group, although not all necessarily significant, were also found between spatial visualization ability measures and the WJ-III ACH Applied Problems subtest (see Table 4).

Spatial Visualization Ability and Visual Imagery

Overall, on the MPI, students used visual images on more than half the problems (8.18 of the 13 problems scored). Using ANOVA, a significant effect for group was found, $F(2, 63) = 3.57$, $p < .05$, $\eta^2 = .10$. Tukey's post hoc procedure revealed a significant difference between G students and students

TABLE 2
Means and Standard Deviations and ANOVA Results for Mathematical Problem-Solving, Visual Imagery, and Spatial Visualization Measures by Group

Measure	LD		AA		G			
	M	*SD*	*M*	*SD*	*M*	*SD*	F^b	Post hoc
MPI								
Total score[a]	2.46	2.26	3.50	2.35	8.45	2.30	42.58***	LD = AA < G
Number of visual images	7.00	3.25	7.68	4.03	9.64	2.80	3.57*	LD < G; AA = LD; AA = G
Pictorial imagery	5.36	2.59	4.45	2.89	2.86	2.32	5.17*	LD > G; AA = LD; AA = G
Schematic imagery	1.64	1.50	3.22	2.98	6.77	3.29	20.80***	LD = AA < G
MGMP-SVT	10.27	5.16	8.50	3.75	16.23	5.34	15.64***	LD = AA <G
WISC-III Block Design	35.36	15.27	39.32	12.87	50.00	11.61	7.09**	LD = AA <G

Note. *N* = 22 for all groups. LD = students with learning disabilities; AA = average achievers; G= gifted students; MPI = *Mathematical Processing Instrument* (Hegarty & Kozhevnikov, 1999); MGMP-SVT = *Middle Grades Mathematics Project Spatial Visualization Test* (Lappan et al., 1982); WISC-III = *Wechsler Intelligence Scale for Children,* 3rd ed. (Wechsler, 1991).
[a]number of problems solved correctly (out of 13). [b]*df* = (2, 63) for all measures.
*$p < .05$. **$p < .01$. ***$p < .001$.

TABLE 3
Correlations Between Measures of Mathematical Problem Solving, Visual Imagery Use, and Spatial Visualization Ability for All Participants

Measure	1	2	3	4	5
1. MPI Math Problem Solving	—				
2. MGMP-SVT	.68**	—			
3. WISC-III Block Design	.55**	.59**	—		
4. MPI Visual Imagery	.38**	.16	.24	—	
5. WJ-III ACH Applied Problems	.85**	.68**	.54**	.33**	—

Note. MPI = *Mathematical Processing Instrument* (Hegarty & Kozhevnikov, 1999); MGMP-SVT = *Middle Grades Mathematics Project Spatial Visualization Test* (Lappan et al., 1982); WISC-III = *Wechsler Intelligence Scale for Children,* 3rd ed. (Wechsler, 1991); WJ-III ACH = *Woodcock-Johnson Tests of Achievement,* 3rd ed. (Woodcock, McGrew, & Mather, 2001).
**$p < .01$.

TABLE 4
Correlations Between Spatial Visualization Ability and Mathematical Problem-Solving Measures by Group

Measure	MGMP-SVT			WISC-III Block Design		
	LD	AA	G	LD	AA	G
MPI Math Problem Solving	.72**	.66**	.25	.50*	.10	.54***
WJ-III ACH Applied Problems	.36	.72**	.61**	.35	.13	.62**

Note. LD = students with learning disabilities; AA = average achievers; G= gifted students; MGMP-SVT = *Middle Grades Mathematics Project Spatial Visualization Test* (Lappan et al., 1982); WISC-III = *Wechsler Intelligence Scale for Children,* 3rd ed. (Wechsler, 1991); MPI = *Mathematical Processing Instrument* (Hegarty & Kozhevnikov, 1999); WJ-III ACH = *Woodcock-Johnson Tests of Achievement,* 3rd ed. (Woodcock, McGrew, & Mather, 2001).
*$p < .05$. **$p < .01$. ***$p < .001$.

with LD. Gifted students used visual images significantly more often than students with LD. No significant differences were found between G and AA students or between AA students and students with LD. Means and standard deviations on these measures are presented in Table 2. The relationships between the use of visual imagery and the two spatial visualization measures are presented in Table 3. Although not significant for either measure, the relationships were positive.

The relationships between the use of pictorial and schematic images and spatial visualization ability on the two measures were also examined. On average, students generated more pictorial images than schematic images for the 13 problems on the MPI. Using ANOVA, significant differences were found among the groups for pictorial imagery, $F(2, 63) = 5.17$, $p < .05$, $\eta^2 = .14$, and schematic imagery, $F(2, 63) = 20.80$, $p < .001$, $\eta^2 = .40$. Tukey's post hoc procedure revealed a significant difference between students with LD and G students for pictorial imagery use. No significant differences between G and AA students or between students with LD and AA students were found. Students with LD used pictorial images significantly more often than G students. For schematic imagery, Tukey's post hoc procedure revealed a significant difference between students with LD and G students and between AA students and G students. Gifted students used schematic images significantly more often than students with LD and AA students. No significant difference between students with LD and AA students was found. Means and standard deviations on these measures are presented in Table 2.

The use of schematic imagery was negatively and significantly correlated with pictorial imagery, $r(64) = -.37$, $p < .01$. Schematic imagery was positively and significantly correlated with each spatial visualization measure. Pictorial imagery was negatively related to each spatial visualization measure; however, this finding was significant only on the MGMP-SVT. The correlations between visual imagery use and spatial visualization measures are presented in Table 5. Examinations of schematic imagery by group revealed the same pattern (see Table 5). For each group, schematic imagery was positively correlated with the spatial visualization measures; however, these correlations were not significant for any measure. Pictorial imagery was negatively and significantly correlated for G students with the MGMP-SVT. Although not significant, correlations were negative for each measure for students with LD and G students. Pictorial imagery for AA students on each measure, however, was positive, although not significant.

Discussion

Spatial visualization ability has been found to correlate with mathematics achievement in the range of .30 to .60 (Ben-Chaim, Lappan, & Houang, 1988; Fennema & Tartre, 1985; Harris, 1981). Spatial visualization skills have been suggested to be important in mathematics, in particular for geometry and for solving complex word problems (Brown & Wheatley, 1997; Burnett, Lane, & Dratt, 1979; Geary, 1996; Wheatley, 1990). Like Hegarty and Kozhevnikov (1999), this study found significant and positive correlations between each spatial visualization measure and mathematical word problem-solving performance on the MPI. Moreover, significant and positive correlations were found between the WJ-III ACH Applied Problems subtest and each spatial visualization measure. Analysis by group of the correlations between each spatial visualization measure and mathematical word problem-solving performance on the MPI and WJ-III ACH Applied Problems subtest, although not significant in all cases, revealed a similar pattern. Overall, students who performed well on the mathematical problem-solving measures performed well on the spatial visualization measures.

On the MPI, gifted students performed significantly better than students with LD and average-achieving students. On average, students with LD obtained a score of 2.5 out of 13 problems solved correctly, and the average-achieving students 3.5 out of 13 problems. Geary (1996) noted that many students find two-step problems considerably more difficult to solve than one-step problems. Similarly, these findings suggest that students with LD and average-achieving students had difficulty solving the mathematical word problems on the MPI. Certainly, research on math problem solving has often demonstrated that many students with LD are poor problem solvers (Miller, Butler, & Lee, 1998; Montague, 1996; Parmar, Cawley, & Frazita, 1996). For each spatial visualization measure, gifted students also performed significantly better than students with LD and average-achieving students. Students with LD and average-achieving students did not differ significantly on the measures of spatial ability. The lower performance of the students with LD on the spatial visualization ability tasks supports the findings of other studies in which students with LD typically perform less well on mathematical tasks that involve some spatial component than their higher achieving peers (Grobecker & De Lisi, 2000; Thornton, Langrall, & Jones, 1997; Wansart, 1990). For the MPI and the measures of spatial visualization ability, the average-achieving students in this sample did not statistically differ from the students with LD. This may suggest that the average achievers had difficulty on these measures or, alternatively, that the students with LD performed higher than expected for both measures.

TABLE 5
Correlations Between Spatial Visualization Measures and Pictorial and Schematic Imagery Use by Group and for All Participants

	Imagery Use	
Measure	Pictorial	Schematic
MGMP-SVT		
Total sample	−.35**	.45**
LD	−.23	.06
AA	.16	.24
G	−.56**	.2
WISC-III Block Design		
Total sample	−.14	.36**
LD	−.11	.02
AA	.26	.20
G	−.08	.14

Note. LD = students with learning disabilities; AA = average achievers; G= gifted students; MGMP-SVT = *Middle Grades Mathematics Project Spatial Visualization Test* (Lappan et al., 1982); WISC-III = *Wechsler Intelligence Scale for Children*, 3rd ed. (Wechsler, 1991).
$^{**}p < .01$.

To solve the problems on the MPI, all students, on average, reported using visual images for approximately 8 out of a possible 13 problems. Gifted students used significantly more visual images than students with LD. Like Hegarty and Kozhevnikov (1999), this study found no significant correlations between the use of visual images and each spatial visualization measure.

Visual images could be classified as being either primarily schematic (i.e., images that encoded the spatial relations described in the problem) or primarily pictorial (i.e., images that encoded objects or persons described in the problem). In examining the relationships between schematic imagery, pictorial imagery, and spatial visualization ability, it was found overall that the use of schematic imagery was positively and significantly correlated with both spatial visualization measures. In contrast, the use of pictorial imagery, although only significant for one measure, was negatively correlated. These findings support the findings of Hegarty and Kozhevnikov's (1999) research. Examination by group performance revealed a similar pattern with the exception of the average-achieving students. Overall, these results suggest that students with low spatial visualization ability showed a preference to use pictorial imagery—that is, a less sophisticated type of imagery. Students with a high spatial visualization ability show a preference for using schematic imagery—a more sophisticated type of imagery—while solving mathematical word problems. Geary (2004) noted the importance of visual–spatial competencies (i.e., the ability to represent forms of conceptual knowledge and to manipulate mathematical information that is cast in spatial form) in solving complex word problems. As Smith (1964) noted, spatial visualization skills may play a central role in problem solving, as the process of abstraction of information from the problem may involve, to some degree, the reproduction of a pattern or structure. Deficits in visual–spatial competencies may interfere with the ability to solve word problems. For example, Zorzi, Prifits, and Umiltá (2002, cited in Geary, 2004) found that participants with a deficit in spatial orientation also demonstrated a deficit in the ability to generate and use a mental number line.

Limitations

Although several notable findings were revealed in this study, some limitations need to be acknowledged. First, the number of mathematical word problems presented to the students was small. Some of the problems may have promoted the use of one type of imagery more than the other type. Furthermore, the students may have used visualization as a strategy more often than they would have had they not been questioned about whether they used visualization to solve the problems. The images generated may have been generated simply as a result of the prompt and not been well thought through, and, as a result, some of the images may have been of poor quality.

Another limitation relates to the problems on the MPI themselves. Although the problems were designed for sixth-grade students, they may have been too difficult for many of the students—in particular the students with LD—who participated in this study. All the problems on the MPI involved two or more steps to solve them and contained complex language structures. It is possible that there was a confounding between the students' understanding of the problem and their resulting imagery. How well they understood the problems is not known. Replication of this study using simpler word problems is recommended. In theory, at least the average-achieving and gifted students should have been able to solve the problems. It should be noted that the researcher was more interested in how the students arrived at their solutions than in whether the solutions were correct. What the findings of this study do not suggest is that the use of schematic imagery and high spatial visualization ability will necessarily result in improved mathematical problem solving. Further research (e.g., an intervention study that involves schematic instruction) is recommended. Moreover, the correlations calculated by group need to be interpreted with some caution, as the total number of participants per group was small. More research with larger samples of students examining the imagery types and spatial visualization ability for the various levels of problem-solving ability is needed.

Finally, not all the students identified as having LD in this study may have had a specific mathematics learning disability. The students in this study, according to their performance on the WJ-III ACH subtests and Broad Math score, represented three levels of mathematics achievement. Clearly, a closer examination is needed. Furthermore, performance on reading and language measures should also be considered. Examination and comparison of the mathematical problem-solving performance, spatial visualization ability, and visual imagery use of students with mathematical disabilities and of students with reading disabilities may help to further clarify the nature of mathematical learning disabilities (Geary, 2004).

Future Research

There is a need for continued investigation of the processes and strategies that good math problem solvers use (Geary, 2004). In this study, students with high spatial visualization ability tended to produce images that were primarily schematic in nature, whereas students with low spatial visualization ability tended to produce images that were primarily pictorial in nature. Furthermore, students who had higher scores on the MPI also had higher scores on the spatial visualization measures.

No information regarding the impact of instructional practices (e.g., visualization instruction, spatial tasks) and materials (e.g., textbooks) that the students may have been exposed to in their classrooms was collected in this study—a consideration for future studies. Teachers play an important role in developing a student's ability to represent a word problem (Presmeg, 1986b). Unfortunately, teachers often do not have the necessary knowledge or understanding of how to teach mathematical problem solving, especially to students with LD. Textbooks rarely provide sufficient instruction on how to solve mathematical word problems. Instruction is usually restricted to textbook models, which often provide a menu of strategies (e.g., draw a diagram, guess, and check). These instructions are not very helpful for students—in particular for students with LD, who often have a difficult time deciding what strategy to use or how to implement the chosen strategy (Fuchs & Fuchs, 2001; Woodward & Montague, 2002). Hegarty and Kozhevnikov (1999) suggested that instruction needs to go beyond getting students to try to "visualize" the problems. We know that involving visual–spatial representations as a component of instruction does seem to improve math problem solving for students with LD (Hutchinson, 1993; Jitendra & Hoff, 1996; Marsh & Cooke, 1996; Zawaiza & Gerber, 1993). Certainly, students need to be taught how to understand and represent the relational elements of mathematics (Van de Walle, 1998). It has been suggested that spatial training may help students to be able to organize the problem with mental pictures, as problems might be better understood through a spatial format (Bishop, cited in Tartre, 1990). Studying the specific roles that spatial visualization skills may have in doing mathematics word problem solving may provide additional insight into this relationship and into developing instructional programs to address specific skill deficits for solving mathematical word problems (Tartre, 1990).

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