whether we choose the normal vector to point out of the cylinder or into the cylinder. Of course, in the inward pointing case, the curvature has the opposite sign, $k = -1/r$.

For inward pointing normals, the formula breaks down when $h > r$, since we get multiple coverage of points in space by points of the form $y + t\nu(y)$.

3. Y is a sphere of radius R with outward normal, so Y_h is a spherical shell, and

$$\begin{aligned} V_3(Y_h) &= \frac{4}{3}\pi[(R+h)^3 - R^3] \\ &= h4\pi R^2 + h^2 4\pi R + h^3\frac{4}{3}\pi \\ &= hA + h^2\frac{1}{R}A + h^3\frac{1}{3R^2}A \\ &= \frac{1}{3}\cdot A\cdot\left[3h + 3\frac{1}{R}\cdot h^2 + \frac{1}{R^2}h^3\right], \end{aligned}$$

where $A = 4\pi R^2$ is the area of the sphere.

Once again, for inward pointing normals we must change the sign of the coefficient of h^2 and the formula thus obtained is only correct for $h \leq \frac{1}{R}$.

So in general, we wish to make the assumption that h is such that the map

$$Y \times [0,h] \to \mathbf{R}^n, \quad (y,t) \mapsto y + t\nu(y)$$

is injective. For Y compact, there always exists an $h_0 > 0$ such that this condition holds for all $h < h_0$. This can be seen to be a consequence of the implicit function theorem. But so not to interrupt the discussion, we will take the injectivity of the map as an hypothesis, for the moment.

In a moment we will define the notion of the various averaged curvatures, $H_1, \dots, H_{n-1}$, of a hypersurface, and find for the case of the sphere with outward pointing normal, that

$$H_1 = \frac{1}{R}, \quad H_2 = \frac{1}{R^2},$$

while for the case of the cylinder with outward pointing normal that

$$H_1 = \frac{1}{2r}, \quad H_2 = 0,$$

and for the case of the planar region that

$$H_1 = H_2 = 0.$$

We can thus write all three of the above the above formulas as

$$V_3(Y_h) = \frac{1}{3}A\left[3h + 3H_1h^2 + H_2h^3\right].$$